

## 12-th Italian Mathematical Olympiad 1996

1. Among the triangles with one side  $l$  and the area  $S$ , find those with the maximum possible product of the three altitudes.
2. Prove that the equation  $a^2 + b^2 = c^2 + 3$  has infinitely many integer solutions  $(a, b, c)$ .
3. Let  $A$  and  $B$  be opposite vertices of a unit cube. Find the radius of the sphere with center inside the cube, tangent to the three faces meeting at  $A$  and to the three edges meeting at  $B$ .
4. Find the number of words of  $n$  letters from the alphabet  $\{a, b, c\}$  which contain an even number of  $a$ 's.
5. Let  $A$  be a point outside a circle  $\mathcal{C}$ . For each point  $P$  on  $\mathcal{C}$ , a square  $APQR$  is constructed, with the vertices  $A, P, Q, R$  in counterclockwise order. Find the locus of  $Q$  as  $P$  runs over  $\mathcal{C}$ .
6. What is the minimum number of squares that one needs to draw on a blank sheet in order to obtain a complete grid  $n \times n$ ?